DE MORGAN

STATEMENT IN ANSWER
TO AN ASSERTION MADE
BY SIR WILLIAM
HAMILTON

S T A T E M E N T

IN ANSWER TO AN ASSERTION

MADE BY

SIR WILLIAM HAMILTON, Bart.,

Professor of Logic in the University of Edinburgh ;

By AUGUSTUS DE MORGAN, *Sec. R.A.S., F.C.P.S.,*

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If any one should be surprised at my printing this statement, on account of the smallness of the pretext for the things alleged against me, I have only to tell him that I am as well aware of the insignificance of that pretext as he is, and that the cogent necessity, as I believe it to be, for taking immediate notice of it, arises from the unquestioned literary celebrity and private worth of a gentleman who was rash enough to tax me with conduct which, could his words be made good, ought to drive me from the society of honest men. The learning and character of the accuser, while they force this publication, make me desirous that it should contain the least possible amount of counter-charge, and cause the least possible annoyance.

This statement arises out of an assertion made by Sir William Hamilton*, Professor of Logic in the University of Edinburgh. He charges me with wilfully and knowingly (for so I interpret the words *breach of confidence* and *false dealing*) attempting to appropriate as my own certain ideas on the subject of logic, which he avers he communicated to me privately. It is true that he has desired to withdraw the letter in which these imputations were made. But this, as will be pointed out, was done in a manner which amounts only to a proposal of suspension followed by private inquiry, and that after a month's persistence in the charge. I prefer a public inquiry. To the assertion I reply, *first,* that the substance of all which I have claimed was deposited at Cambridge, and out of my hands, before I ever saw his handwriting ; *secondly,* that no *intelligible* communication was ever made by him to me ; *thirdly,* that so soon as I received anything which induced me to suppose that we must have had something in common, I gave him full information of what I then had by return of post, even to the means of identifying the papers on which all that

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* To prevent all mistake, I must desire the reader to avoid a confusion which frequently occurs between the eminent professor of logic in the University of Edinburgh, and my friend Sir William *Rowan* Hamilton of Dublin, Astronomer Royal for Ireland, and so widely known as one of the first of our mathematicians.

development was written which was not already at Cambridge; *fourthly*, that I have ~~strong~~ subsequent reason to suppose we really have nothing in common; *fifthly*, that his mode of conducting his own case, subsequently to the production of his charge, is enough to satisfy any reasonable person that he never had any confidence in his own power of proving it.

I shall offer proof of the first, second, and third of the above points. The fourth I can so far establish as to throw upon Sir William Hamilton the onus of proving that he ever published in his lectures or otherwise any of the notions which I have advanced; indeed, he must in any case furnish this proof. The fifth is matter of opinion, which I must leave to the reader.

There is something to say which is not evidence, except to those who know me. I think I can show that I never had the *opportunity* of copying Sir William Hamilton's ideas; but, be this how it may, I declare upon my honour that I never derived a single hint of any kind from his communications. I declare also, that I remember (and what is more to the purpose, that I remembered in February, when I printed it,) that the papers which I gave the means of identifying in January, were written before I received any communication from Sir William Hamilton, except a civil note, promising to answer certain questions on the *history* of logic when he returned to Edinburgh. In any question of mere priority which may arise, these papers of course can only date, at the earliest, from the time at which I announced their existence. But as regards my own integrity in this matter, the two declarations above made will, as I said, be enough with those who know me. I will now proceed with my statement.

Early in the last summer I began to collect matters, which had suggested themselves to me at different times, connected with the theory of the syllogism in logic. My intention was, and is, to enlarge a slight treatise on that subject, which I published a few years ago, into a work on formal logic. In the course of my investigations I fell upon various matters which I thought worth communicating to the Cambridge Philosophical Society. I accordingly sent to that body a Memoir headed, 'On the Structure of the Syllogism, and on the Application of the Theory of Probabilities to questions of Argument and Authority.' That paper is now printed, and will appear in vol. viii. part 3 of the Cambridge Transactions. It is dated *October* 3, 1846. The Master of Trinity College (to whom I sent it in the first instance, and from whose hands it passed into those of the Secretary of the Society) testifies that he received it on the 6th. The manuscript, and the first proof taken from it, are in existence and can be produced; neither has any corrections worth noting. By comparison of the dates of my communications from Sir William Hamilton (the first is dated *October* 7, and the first professing to speak of logic is of *November* 2), it will be seen that I have no occasion to say any more about the body of the paper.

But when the printed proof came into my hands in February last, I made an *addition* at the end. This addition I assert to be a mere development of what is in the body of the paper; and on the truth of this the question turns whether I *can* have borrowed, knowingly or unknowingly, from Sir William Hamilton. If there *be* any thing new in principle in the addition, then comes the question whether Sir William Hamilton's communication *could* have given me a hint of *that new principle*. To enable those who can and will to judge for themselves, I shall presently print—

1. The part of my paper which I assert to contain the principles developed in the addition. 2. The only communication which Sir William Hamilton can make a pretext for his charge, as will distinctly appear in my statement

of our correspondence. 3. All* the addition to my paper which was in any way mentioned in those letters of mine to Sir William Hamilton on which he wrote his charge.

But as those who are not inclined to go into such matters will judge by the manner in which the imputation was made and met, I begin by a statement of all that has taken place between us.

Sir William Hamilton of Edinburgh stands very high among the culti- vators of mental philosophy in these kingdoms, and is well known to have paid profound attention to the history of that science in all ages. Of his character as a man I never heard anything but the highest praise ; and I most distinctly say, that I am perfectly satisfied he believed the charge when he made it, and that, more or less doubtfully, he believes it up to this moment. At the end of September I wrote to him, without previous acquaintance, to gain some information on existing sources as to the *history* of technical logic. I think I mentioned my own investigations in general terms. The answer, dated *Oct.·7* (received by me four days after my paper had been sent to Cam- bridge), contains the following :—

" I have for many years taught in my class, what, I think, affords a full extension and, thereby, a complete simplification, of the syllogistic theory ; and through the Notes and Essays of my students, this development of the doctrine has obtained considerable publicity, though I have not yet given it to the world through the press. Should you feel any curiosity on this matter ; or should you wish for a list of the professed Histories of Logic, I will do what I can to satisfy you, on my return home, in about a fortnight hence, ..."

In my reply, I accepted Sir William Hamilton's offer as to his system of syllogism. The next communication I received was dated *Nov.* 2 ; it is (why will presently appear) the pretext for the charge made against me, and is printed in the sequel *entire*, between the two extracts from my paper. I shall reserve my comment on it, and shall point out the reason why it was as unintelligible to me as I predict it will be to the reader. I replied to this letter, with some remarks on parts of the *requisites* for the prize essay. The next communication is dated *Dec.* 28, and consisted of—1. A letter. 2. A printed prospectus of Sir William Hamilton's intended work on logic. Nothing turns on this, for the simple reason that my answer contained the most express and formal proof that, come by it how I might, I was *then* in the most com- plete *written* possession of all I have since published. The following is an extract from the letter :—

" I send you what I promised—a more articulate statement of my syllogistic theory. I should have made it however less summary, had the prize Essays (for which I sent you the Requisites) been given in ; but the day of delivery is not till the 1st of January. I have ap- pended this statement to my edition of Reid's Works, which has just appeared, and should have sent it to you sooner, but waited for an answer to my application to Mr. —— ——, which ought to have come some ten days earlier. That gentleman was a pupil of mine six years ago, and obtained one of the highest honours of the class ; he was therefore fully competent to afford you information, which I begged † him to do, in regard to my logical doctrines

* To the best of my knowledge and belief: for I kept no copies of my letters previous to March 13. I have no doubt Sir William Hamilton would have supplied them ; but I did not think it necessary to ask, as it is on *his* communications that the charge was made.

† Since that time, Mr. —— ——, who was once my own pupil, called on me one evening after a lecture at University College, and left with me his notes of Sir William Hamilton's lectures. My knowledge of these notes has nothing to do with the present question, since it dates from some weeks after the 1st of January. I shall allude to them again presently. Not to omit mention of any communication which has passed, I should state that Sir William Hamilton, about the time I received these notes, sent me part of his supplement to Reid, con- taining notes into which an enormous amount of digested learning is thrown, but not having

proceed briefly to answer your communications of the 31st Dec. and 1st Jan. ult., which reached me after I was laid up by a tedious inflammatory attack. From these (if I am not mistaken) it appears that you claim for yourself the independent re-discovery of the funda-mental doctrine of syllogism, which I privately communicated to you, and of many of its most important consequences more fully developed in the printed prospectus. This claim, though it be only to secondary originality, I am altogether unable to admit. To me, it is manifest, that for the principle of the doctrine you are wholly indebted to my information ; and I cannot but think that if you (though recognising always my priority) give forth that doctrine as a speculation of your own, you will be guilty,—pardon the plain speaking,—both of an injurious breach of confidence towards me and of false dealing towards the public. Am I therefore correct in my understanding of your letters ? I shall be glad to find that I am wrong. "I remain, dear Sir, your most obedient Servant, "W. HAMILTON."

There is nothing so pardonable as plain speaking ; in the present instance I owe to it the opportunity of strangling this imputation in its cradle. I replied by return of post, that I saw the propriety of abstaining from fur-ther private correspondence upon the subject in question ; that, when my paper was published, he should have some copies ; and that the hasty manner in which he had expressed his suspicions of an odious charge did not di-minish the respect in which I held him. In a few days I received copies of my paper in the Cambridge Transactions, of which I forwarded a few copies to Edinburgh in a bookseller's parcel. On the 25th of March, judging they had been received, I addressed Sir William Hamilton again, still in terms of respect (indeed, I never have used any others), informing him that I intended to bring his charge to a speedy, and if necessary, a public issue; and giving him till the 10th of April (or any not unreasonably later date which he should propose) either to retract, or to announce the time and manner in which he would maintain his assertions by public proof: failing his acceptance of either alternative, I informed him that I should draw up a statement myself. The answer, dated the 27th, informed me that my paper had not been received, but that the expressions bore exclusive reference to my letters, and that there was no occasion to wait. That (I quote descriptively, in the third person) it would afford him sincere pleasure to be enabled to retract what was written both painfully and unwillingly, but which he could not but write under the con-viction of its truth (if he had said, under the conviction of his ability to prove it, it had been more to the purpose). That there were only two ways in which this retractation would be for him possible : he must be satisfied either that the meaning of my letters was not what he supposed them unambiguously to express; or that I had been acting under the influence of some intellectual error (that he himself should be in such a predicament is virtually declared impossible, except only by misunderstanding my letters). That it would truly gratify him to be allowed, on either alternative, to cancel all derogatory expres-sions and sentiments towards one whom otherwise, on many accounts, he was so much inclined to respect. That he cordially sympathised with my feelings on the occasion (I wonder how ; being sure that he believed his own charge, I should have thought he would have taken me to be putting a false front of in-tegrity over a consciousness of wilful theft): and that assuredly his co-opera-tion should not be wanting to lay the *whole* relevant documents fairly and faith-fully before the public,—on the supposition, always, that my book preferred the same claim as my correspondence. That I might depend on a statement from him, of the grounds on which he felt himself constrained to advance the obnoxious allegations ; and that if his health continued as it was, I should receive it (*in manuscript, mind*) before the time I specified. But, as his public statement was thus to be communicated to me, of course mine, in like manner, would be communicated to him, before their joint publication ; and

either, each made independent of the other, and final; or, if I were allowed to answer him, he should equally be allowed to answer me. He would therefore, he concluded, before writing, await my determination upon this point.

If I had felt any anger at the hasty and utterly groundless imputation which has given me the trouble of making this statement, I think the preceding letter would have removed it. There is something racy about the idea of private declaration and answer before going into court, circumstanced as we were. In cases of amicable literary controversy it would be an excellent plan, and would save a good deal of printing. But the notion of the prosecutor and the accused thief quietly interchanging their notes of evidence, and settling its balance before trial, would, if carried into effect, have vastly tickled the public, never indisposed to find jokes in controversies of this sort. I replied, *March* 30, by totally declining the arrangement, and calling upon Sir William, as he valued his own honour, either forthwith to retract, or publicly to attempt the proof of his accusation. I begged him to understand distinctly, that when I proposed to him to retract, it was because such a mark of consideration was usual and proper : but that as far as I was concerned, I had much rather that he should attempt publicly to maintain his words. I added, that I should give no explanations except in print : that had they been asked for in the first instance I would have given them ; but certainly not to a person who had preferred to make facts of his surmises, and inferences of his suspicions. I now certainly thought I should draw him out.

From that time to the limit fixed, the 10th of April, I heard no more directly from Sir William Hamilton. But, in a circuitous way, I was offered a sort of retractation on terms. Sir William, I am certain, did not believe me when I said that I would rather he should attempt to maintain than retract. A friend of mine, to whom I had mentioned the whole matter, numbers among his friends one of Sir William Hamilton's, a gentleman of high consideration at Edinburgh. To this gentleman my friend wrote, expressing his regret at the quarrel that was taking place. Some correspondence ensued, to which, as I was no party to it (though I heard extracts read), I should not have alluded, had it not been that Sir William Hamilton became a party to it. And in the course of it he suggested (with a full knowledge, I believe, that it would be conveyed to me), that if I could manage to remember that the papers I had described in one of my letters of Dec. 31 and Jan. 1 were not written till *after* I had seen his letter, &c. of November 2, he, on his part, could say that—I forget the words, but the substance was that—I was probably acting only under intellectual confusion, and not under intention to appropriate his ideas. This I simply refused, for two reasons ; I forget whether I gave them or not, but here they are. In the first place, I could not remember what did not happen : though, as I will show presently, it has nothing to do with the matter, yet those papers *were* written before the letter of November arrived. And in the second place, nothing should ever induce me to buy the withdrawal of such a charge by any concession whatever. I could only meet it by every possible defiance, expressed in the most peremptory terms which would consist with the respect I wished (and wish) to preserve towards great learning, high character, and ill health.

When the 10th of *April* had passed over, I thought it necessary to make one more effort. Accordingly, on the 12th, I wrote to Sir William Hamilton, asking first, whether he was prepared to give me his assurance that he would immediately proceed to the public proof of his charge of *March* 13 ; and secondly, when I might expect his publication. I added, that on failure of an immediate and explicit answer to both questions, I should consider him as

shrinking from the attempt to show cause for his assertions, and should pro-
claim him as so shrinking in the public statement which I should immediately
draw up.

On the 15th of *April* I received, dated the 13th, what at first I thought
(the course of post making it possible) was the answer: and what Sir William
Hamilton allowed to stand in place of an answer for several days :—

" DEAR SIR,—As I find that my letter of the 13th March ult. prevents you from favouring
me with an answer to the inquiries which I would beg leave to propose ; and as I am anxious
that there should be no possible misunderstanding in reference to your claim of being an
original exeogitator of the doctrine of syllogism founded on the expressed quantity of the
predicate ; I request your permission to withdraw that letter—to hold it unconditionally *pro
non scripto* ; expressing also my regret that the allegations it contains, though only hypo-
thetical*, were stated before you had an opportunity of explanation. This being done on
my part, I trust on yours, that you will not deny me the satisfaction of answering the ques-
tions which I am desirous to put ; in the hope that the whole question may be solved by the
detection of a mistake on the one side or the other. I remain, dear Sir,

" Your most obedient Servant, " W. HAMILTON."

I acknowledged the receipt of this letter, referring to my statement now
before the reader for the further answer, which I proceed to give.

In the first place, I never will have anything to do with the doctrine of
scriptum pro non scripto. The only Latin I hold by on such a matter is
Litera scripta manet. I understand a man when he says he has changed his
opinion, or that he will not or cannot maintain what he formerly said he could
maintain ; but I do not understand holding what was written as not having
been written. If Sir William Hamilton had said in effect, " I have made a
charge, which, though I thought it true at the time, I now do not believe,"
or " which, though I thought I could prove at the time, I do not find I can
prove it now," he would have put me in a difficult position. I am not at all
prepared to say that even in such a case, I should, looking at all that has
passed, have judged it safe to withhold my present statement. But, with
reference to the letter above given, I am firmly and clearly of opinion, that I
owe this publication to myself, to the Cambridge Society, and to the college
in which I teach. In the first place, the withdrawal is made, not because the
accuser has found out his mistake, but because he finds the accused will not
enter on the matter with him while it stands, except with the whole public to
judge. Secondly, it is a *suspension,* and a *suspension* only : Sir William
Hamilton, who now sees that he came to a decision too hastily, invites me
into the court in which he proposes to *hear* the case. I am to defend myself
upon interrogatories, and a hope is expressed that it will turn out to be all a
mistake. But if that hope should be disappointed (as ten to one it would
be), what must be supposed likely to follow ? a renewal of the charge ; the
suspension will be taken off. Thirdly, the acceptance of a withdrawal of the
charge, accompanied by regret expressed for having made it before I could
explain, would be an admission on my part, that there was some reasonable
primâ facie case against me. Now, if Sir William Hamilton should complain
that I would not give him grounds for retracting his assertion, I ask him,
what were his grounds for making it ? I say he had *none* but what were
utterly futile : I say this before I absolutely know what they were ; and once

* So far hypothetical that they depend on my " book preferring the same claim as my
correspondence," and upon Sir William Hamilton's correctly understanding my letters. Now
the book (that is, the Memoir) *does* prefer that claim ; and my correspondence *does*, as Sir
William Hamilton correctly understands it, make a full declaration of my having indepen-
dently arrived at everything new which I did arrive at. Besides, he afterwards informs me
that he was constrained to write the derogatory expressions under a conviction of their truth.

again I challenge him to produce them, and to let them undergo my public handling.

My reader will see that I have been most anxious to be put on my defence, or as Sir William says, to give explanations—refutations I should call them. But not to my respected accuser himself: I have preferred to appeal to Cæsar; and Sir William would have done better if he had answered, Unto Cæsar shalt thou go.

[This statement was put into the printer's hands on the 19th of April, and on the 23rd I received the following note, dated the 20th, being the seventh day after the receipt of the letter in which I had asked for an immediate and explicit answer to the two questions above:

"Sir William Hamilton presents his compliments to Professor De Morgan, whose note of the 12th he delayed answering, until he received the reply to his own of the 13th. Sir Wm. Hamilton is surprised, that Mr. De Morgan could think the intimation, he required, even possible, so long as the conciliatory intervention of Mr. De Morgan's friend Dr. ——— was not ended, and ended, in effect*, it was not, until Mr. De Morgan's last note terminated all hope of the difference being resolved into a mistake, on the one side or the other: Sir William Hamilton has now therefore no alternative †, but to state the evidence on which he disputes Mr. De Morgan's claim to an independent rediscovery ‡ of the new principle of syllogism; and is too confident of the strength of his position, to deny his opponent the advantage of seeing the one case, before publishing the other. Sir William will, therefore, proceed without delay; and though he has at present much ** else to do, hopes, if health permit him, to have his proof written during the present week, and printed in the course of next. There is however a strike at present among the Edinburgh printers. What effect †† this may have in retarding the publication Sir William Hamilton is unable to say."]

My reader must take notice, that I and my accuser are at issue on some important points of social ethics. He "could not but write," he was "constrained" to write, under "the conviction of the truth" of his charge. In the day of swords, it was one of the objects of public policy to prevent people from sticking them into each other's bodies upon trivial grounds. We now wear pens: and it is just as great a point to hinder ourselves from sticking them into each other's characters without serious and well-considered reasons. To this end, I have always considered it as one of the first and most special rules, that conviction of the truth of a charge is no sufficient reason for its promulgation. I assert that no one is justified in accusing another *until he has his proof ready*: and that, in the interval, if indeed it be right that there should be any interval, between the charge and the attempt at substantiation, all the leisure and energies of the accuser are the property of the accused. Such being my mind, I cannot now accept Sir William Hamilton, whose theory and practice are so much at variance with mine, either as an opponent in private controversy, or a judge of the result of mutual explanations. More than this, there has been throughout his correspondence, too much of the feeling that he could hold certain points as assumed, and upon those assumptions frame his own conditions, without any reference to whether I granted them or not: and too little of the feeling that when an accusation is

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* I was no party to that correspondence, as was avouched by the letters themselves. It was terminated before the 10th of *April*, with a definite announcement, on the part of my friend Dr. ———, that he saw the matter must become public. The distinction between termination in *fact*, and termination *in effect*, is above my comprehension.

† This regretful announcement is curious, as addressed to me. What have I been pressing for all along?

‡ This will not do. *Breach of confidence* and *false dealing* must be maintained, or Sir William Hamilton abandons his charge.

** So have I.

†† It need have none at all. Why not print at Glasgow?

made, the bringer of it puts himself into the balance as well as the party whom he accuses. He is not, I am well aware from his writings, apt to think much of the mathematician out of mathematics : but he must bear with me while I quote, without meaning to apply its point offensively, something for his in struction from the mouth of a mathematician. The late Professor Vince was once arguing at Cambridge against duelling, and some one said, " Well, but, Professor, what could you do if any one called you a liar." " Sir," said the fine old fellow, in his peculiar brogue, " I should tell him to pruv it : and if he did pruv it, I should be ashamed of myself : and if he didn't pruv it, *he* ought to be ashamed of himself." I will not permit myself to suppose that Sir William Hamilton is either ignorant of, or unconsenting to, the infamy which would attach to any one who had deserved the terms he used for the conduct he described. But I doubt if he be aware of the serious character of his own position, if he persist in declining to attempt the establishment of what he has said more than once he believed in his right to assert, and what, up to this moment, he has only suspended the assertion of, with no other avowed purpose except that of inducing me to put myself on my justification for his private satisfaction, instead of proceeding to a public discussion. [And now my doubt is changed into certainty. For in the letter introduced in [   ], Sir William Hamilton grants me permission to see, if I please, the evidence on which I am accused before I publish my answer. Not because it is my right; not because it is due to his own character ; but because he feels himself so strong that he can afford it. It is rather a better presumption for my case, that I feel I can afford to do without it. I will not wait for his tardy publi cation : but, if it ever do appear, I will try the " strength of his position " at once, and the reader shall soon see how I set about it.]

I know how it would be if I allowed the matter to end thus. Sir William Hamilton has by this time no doubt put it out of his own power to silence whispers, those whispers which sometimes circulate for a quarter of a century, and are then published in recollections and memoirs, perhaps at a time when neither party can correct them. Of course *I* have widely announced the pur port of the letter of March 13 : I receive no confidences of that kind. Ac cordingly I am now determined to prevent all risk as to this matter : and further, to show any one who, having the character which will give weight to his words, may be disposed to attack mine, that he will have to look to his own retreat ; that his bridge shall neither be of gold, nor of my providing. I will now proceed to the extracts already promised.

To make the extracts from my paper intelligible, I premise that I consider every proposition as limited, if so mentioned, to a given range of objects of thought ; the whole collection of which I call its *universe*. If no limitation be intended, then the universe of the proposition is *the* universe. *Contrary* names are those one of which must belong to every object of thought, but never both : and they are denoted by large and small letters. Thus x means not-X, and everything is either X or x. And X and Y being two names, and with reference to this order, the following notation is used :—

(A) or X) Y means ' Every X is Y' | (O) or X: Y means 'Some Xs are not Ys'
(E) or X.Y . . . . . . ' No X is Y' | (I) or XY . . . . 'Some Xs are Ys'
(a) or x) y . . . . . . ' Every x is y' | (o) or x:y . . . . 'Some xs are not ys'
(e) or x.y . . . . . . ' No x is y' | (i) or xy . . . . 'Some xs are ys'

When Z is introduced instead of X, the same vowels are used.

Extract I. *From the paper as sent to Cambridge before I had any communica-
tion whatsoever from Sir William Hamilton (without any corrections).*

Section III. *On the quantity of propositions.*

"The logical use of the word *some*, as merely ' more than none,' needs no further expla-
nation. Exact knowledge of the extent of a proposition would consist in knowing, for in-
stance in ' some Xs are not Ys,' both what proportion of the Xs are spoken of, and what pro-
portion exists between the whole number of Xs and of Ys. The want of this information compels
us to divide the exponents of our proportions into 0, more than 0 not necessarily 1, and 1.
An algebraist learns to consider the distinction between 0 and quantity as identical, for many
purposes, with that between one quantity and another : the logician must (all writers imply)
keep the distinction between 0 and a, however small a may be, as sacred as that between 0
and $1-a$: there being but the same form for the two cases. We shall now see that this matter
has not been fully examined.

"Inference must consist in bringing each two things which are to be compared into com-
parison with a third. Many comparisons may be made at onee, but there must be this process
in every one. When the comparison is that of identity, of *is* or *is not*, it can only be in its
ultimate or individual case, one of the two following:—' This X is a Y, this Z is the very
same Y, therefore this X is this Z ; or else ' This X is a Y, this Z is not the very same Y,
therefore this X is not this Z.' And collectively, it must be either ' Each of these Xs is a Y ;
each of these Ys is a Z ; therefore each of these Xs is a Z ;' or else ' Each of these Xs is a
Y, no one of these Ys is a Z, therefore no one of these Xs is a Z.'

" All that is essential then to a syllogism is that its premises shall mention a number of Ys,
of each of which they shall affirm either that it is both X and Z, or that it is one and is not the
other. The premises may mention more : but it is enough that this much can be picked out ;
and it is in this last process that inference consists.

" Aristotle noticed but one way of being sure that the same Ys are spoken of in both pre-
mises : namely, by speaking of all of them in one at least. But this is only a case of the rule :
for all that is necessary is *that more* Ys *in number than there exist separate* Ys *shall be spoken
of in both premises together.* Having to make $m+n$ greater than unity, when neither m nor
n is so, he admitted only that case in which one of the two m or n, *is* unity and the other is
anything except 0. Here then are two syllogisms which ought to have appeared, but do not,

Most of the Ys are Xs	Most of the Ys are Xs
Most of the Ys are Zs	Most of the Ys are not Zs
∴ Some Xs are Zs	∴ Some of the Xs are not Zs.

And instead of most, or $\frac{1}{2}+a$, of the Ys, may be substituted any two fractions which have a
sum greater than unity. If these fractions be m and n, then the middle term is *at least* the frac-
tion $m+n-1$ of the Ys. It is not really even necessary that all the Ys should enter in one pre-
miss or the other : for more than the fraction $m+n-1$ of the whole may be repeated twice.

" And in truth it is this mode of syllogising that we are frequently obliged to have recourse
to ; perhaps more often than not in our universal syllogisms. ' *All* men are capable of some
instruction ; all who are capable of any instruction can learn to distinguish their right and left
hands by name ; therefore all men can learn to do so.' Let the word *all* in these two cases
mean only *all but one*, and the books on logic tell us with one voice that the syllogism has
particular premises, and *no conclusion can be drawn*. But in fact idiots are capable of no in-
struction, many are deaf and dumb, some are without hands : and yet a conclusion is admis-
sible. Here m and n are each very near to unity, and $m+n-1$ is therefore near to unity.
Some will say that this is a probable conclusion : that in the case of any one person it means
there is the chance m that he can receive instruction, and n that one so gifted can be made to
name his right and left hand : therefore $m \times n$ (very near unity) is the chance that this man
can learn so much.

" But I cannot see how in this instance the probability is anything but another sort of in-
ference from the demonstrable conclusion of the syllogism, which must exist under the pre-
mises given. Besides which, even if we admit the syllogism as only probable with regard to any
one man, it is absolute and demonstrative in regard to the proposition with which it concludes.

" But this is not the only case in which the middle term need not enter universally : this
however is matter for the next Section. I now go on to another point."

Extract II.

" I now take the two cases in which particular premises may give a conclusion : namely

$$I_{II} \qquad XY+XY=XZ \qquad\qquad XY+Y:Z=X:Z \qquad O_{Io}$$

on the suppositions that the Ys mentioned in both premises are in number more than all the

Ys. If Y_1 and Y_2 stand for the fractions of the whole number of Ys mentioned or implied in the two premises, and y_1 and y_2 for the fractions of the *ys* implied or mentioned, we shall by a repetition of the process on $YX + YZ = XZ$ (the other being obtained in the course of the process) arrive at the following results or their counterparts : remembering that $Y_1 + Y_2$ is greater or less than 1, according as $y_1 + y_2$ is less or greater.

Designation.	Syllogism.	Condition of its existence.
I_{II}	$YX + YZ = XZ$	$Y_1 + Y_2$ greater than 1
O_{Io}	$YX + Y : Z = XZ$	....
i_{oo}	$Y : X + Y : Z = xz$	....
O_{oi}	$X : Y + yz = X : Z$	$Y_1 + Y_2$ less than 1
i_{ii}	$yx + yz = xz$	....
O_{Oi}	$X : Y + yz = X : Z$	....
I_{OO}	$X : Y + Z : Y = X Z$	........................"

Communication received on the 4th or 5th of November from Sir William Hamilton, being the pretext for his charge that I have, with injurious breach of confidence towards himself, and false dealing towards the public, appropriated his " Fundamental Doctrine of Syllogism" privately communicated to me.

" 16 Great King Street,
November 2nd, 1846.

" DEAR SIR,—I have been longer than I anticipated in answering your last letter. I now send you a copy of the requisites for the prize Essay, which I gave out to my students at the close of last session. It will show you the nature of my doctrine of syllogism, in one of its halves. The other, which is not there touched on, regards the two wholes, or quantities in which a syllogism is cast. I had intended sending you a copy of a more articulate statement which I meant, at any rate, to have drawn up; but I have not as yet been able to write this. I will send it when it is done. From what you state of your system having ' little in common with the old one,' and from the contents of your First Notions, we shall not, I find, at all interfere, for my doctrine is simply *that of Aristotle, fully developed.*

It will give me great pleasure if I can be of any use, in your investigations concerning the history of Logical doctrines. I have paid great attention to this subject, on which I found, that I could obtain little or no information from the professed historians of Logic ; and my collection of Logical books is probably the most complete in this country. But, as I mentioned to you in my former letter, it is only in subordinate matters that in *abstract* Logic there has been any progress.   " I remain, dear Sir, very truly yours,

" W. HAMILTON."

Essay on the new Analytic of Logical Forms.

Without wishing to prescribe any definite order, it is required that there should be stated in the Essay,—

1°. What Logic *postulates* as a condition of its applicability.

2°. The reasons why common language makes an *ellipsis* of the *expressed quantity*—frequently of the *subject*, and more frequently of the *predicate*, though both have always their quantities in thought.

3°. *Conversion of propositions*—on the *common* doctrine.

4°. Defects of this.

5°. *Figure* and *Mood* of Categorical syllogism, and Reduction,—on *common* doctrine (General statement).

6°. Defects of this (General statement).

7°. The one *supreme Canon* of Categorical Syllogisms.

8°. The evolution, from this canon, of all the *species* of Syllogism.

9°. The evolution, from this canon, of all the *general laws* of categorical Syllogisms.

10°. The error of the *special laws* for the several Figures of Categorical Syllogism.

11°. *How many Figures* are there.

12°. What are the *Canons* of the *several Figures.*

13°. *How many moods* are there in all the Figures : showing in concrete examples, through all the Moods, the *unessential* variation which Figure makes in a syllogism.

(14° was missing, or those which follow 13° were wrong numbered.)

15°. What relation do the Figures hold to *extension* and *comprehension.*

16°. Why have the *second* and *third* Figures *no determinate major* and *minor* premises and *two* indifferent conclusions ; while the *first* Figure has a *determinate major and minor premise,* and a single *proximate conclusion.*

17°. What relation do the Figures hold to *Deduction* and *Induction*.

N.B. This Essay open for competition to all students of the class of Logic and Metaphysics during the last or during the ensuing session.

April 15th, 1846.

Extract from the Addition to my Paper, taken, as can be shown, from the papers which I gave the means of identifying in January last, and which papers (though I hold it immaterial) I assert to have been written before I received any logical communication from Sir William Hamilton. (To be compared with the extracts given above.)

" Since this paper was written, I found that the whole theory of the syllogism might be deduced from the consideration of propositions in a form in which *definite quantity* of assertion is given both to the subject and the predicate of a proposition. I had committed this view to paper, when I learned from Sir William Hamilton of Edinburgh, that he had for some time past publicly taught a theory of the syllogism differing in detail and extent from that of Aristotle. From the prospectus of an intended work on logic, which Sir William Hamilton has recently issued, at the end of his edition of Reid, as well as from information conveyed to me by himself in general terms, I should suppose it will be found that I have been more or less anticipated in the view just alluded to. To what extent this has been the case, I cannot now ascertain; but the book of which the prospectus just named is an announcement, will settle that question. From the extraordinary extent of its author's learning in the history of philosophy, and the acuteness of his written articles on the subject, all who are interested in logic will look for its appearance with more than common interest.

" The footing upon which we should be glad to put propositions, if our knowledge were minute enough, is the following. We should state how many individuals there are under the names which are the subject and predicate, and of how many of each we mean to speak. Thus, instead of ' Some Xs are Ys,' it would be, ' Every one of a specified Xs is one or other of b specified Ys.' And the negative form would be as in ' No one of a specified Xs is any one of b specified Ys.' If propositions be stated in this way, the conditions of inference are as follows. Let the *effective number* of a proposition be the number of mentioned cases of the *subject*: if it be an affirmative proposition, or of the *middle term*, if it be a negative proposition. Thus, in ' Each one of 50 Xs is one or other of 70 Ys,' is a proposition, the effective number of which is always 50. But ' No one of 50 Xs is any one of 70 Ys' is a proposition, the effective number of which is 50 or 70, according as X or Y is the middle term of the syllogism in which it is to be used. Then two propositions, each of two terms, and having one term in common, admit an inference when 1. They are not both negative. 2. The sum of the effective numbers of the two premises is greater than the whole number of existing cases of the middle term. And the excess of that sum above the number of cases of the middle term is the number of the cases in the affirmative premiss which are the subjects of inference. Thus, if there be 100 Ys, and we can say that each of 50 Xs is one or other of 80 Ys, and that no one of 20 Zs is any one of 60 Ys ;—the effective numbers are 50 and 60. And $50 + 60$ exceeding 100 by 10, there are 10 Xs, of which we may affirm that no one of them is any one of 20 Zs mentioned.

" The following brief summary will enable the reader to observe the complete deduction of all the Aristotelian forms, and the various modes of inference from *specific particulars*, of which a short account has already been given.

" Let Q be the whole number of Xs ; and t the number specified in the premiss. Let c be the whole number of Zs ; and w the number specified in the premiss. Let b be the whole number of Ys ; and u and v the numbers specified in the premises of x and z. Let $X_t Y_u$ denote that each of t Xs is affirmed to be one out of u Ys : and $X_t : Y_u$ that each of t Xs is denied to be any one out of u Ys. Let $X_{m, n}$ signify m Xs taken out of a larger specified number n ; and so on. Then the five possible syllogisms, on the condition that no contraries are to enter either premises or conclusion, are as follows :—

1. $X_t Y_u + Z_w Y_v = X_{t+w-b,\ t}$ $Z_w = Z_{t+w-b,\ w} X_t.$
2. $X_t Y_u + Y_v Z_w = X_{t+v-b,\ t}$ $Z_w = Z_{t+v-b,\ w} X_t.$
3. $Y_u X_t + Y_v Z_w = X_{u+v-b,\ t}$ $Z_w = Z_{u+v-b,\ w} X_t.$
4. $X_t Y_u + Z_w : Y_v = X_{t+v-b,\ t} : Z_w.$
5. $Y_u X_t + Z_w : Y_v = X_{u+v-b,\ t} : Z_w.$

" The condition of inference expresses itself; in the $X_{m,\ t}$ of the conclusion, m must neither be 0 nor negative. The first case gives no Aristotelian syllogism ; the middle term never

entering universally (of necessity) into any of its forms, under any degree of specification which the usual modes of speaking allow. The other cases divide the old syllogisms among themselves in the following manner: they are written so as to show that there is sometimes a little difference of amount of specification between the results of different figures, which changes in the reduction from one figure to another. The Roman numerals mark the figures.

2.	$t=a,\ v=b$	$Y)Z_w + X)Y_u = X)Z_{v,w}$	*Barbara* I.
	$t=a,\ v=b$	$X)Y_u + Y)Z_w = Z_{a,w}X$	*Bramantip* IV.
	$t<a,\ v=b$	$Y)Z_w + X_tY_u = X_tZ_{u,w}$	*Darii* I.
	$t<a,\ v=b$	$X_tY_u + Y)Z_w = Z_{t,w}X_t$	*Dimaris* IV.
3.	$u=b,\ v=b$	$Y)X_t + Y)Z_w = Z_{b,w}X_{b,t}$	*Darapti* III.
	$u<b,\ v=b$	$Y_uX_t + Y)Z_w = Z_{u,w}X_{u,t}$	*Disamis* III.
	$u=b,\ v<b$	$Y)X_t + Y_v)Z_w = Z_{v,w}X_{v,t}$	*Datisi* III.
4.	$t=a,\ v=b,\ w=c$	$Y.Z + X)Y_u = X.Z$	*Celarent* I.
	$t-a,\ v=b,\ w=c$	$Z.Y + X)Y_u = X.Z$	*Cesare* II.
	$t=a,\ v=b,\ w=c$	$X)Y_u + Z.Y = Z.X$	*Camestres* II.
	$t=a,\ v=b,\ w=c$	$X)Y_u + Y.Z = Z.X$	*Camenes* IV.
	$v=b,\ w=c$	$Y.Z + X_tY_u = X_t:Z$	*Ferio* I.
	$v=b,\ w=c$	$Z.Y + X_tY_u = X_t:Z$	*Festino* II.
	$t=a,\ v=b,$	$X)Y_u + Z_w:Y = Z_w:X$	*Baroko* II.
5.	$u=b,\ v=b,\ w=c$	$Y.Z + Y)X_t = X_{b,t}:Z$	*Felapton* III.
	$u=b,\ v=b,\ w=c$	$Z.Y + Y)X_t = X_{b,t}:Z$	*Fesapo* IV.
	$v=b,\ w=c$	$Y.Z + Y_uX_t = X_{u,t}:Z$	*Feriso* III.
	$v=b,\ w=c$	$Z.Y + Y_uX_t = X_{u,t}:Z$	*Fresison* IV.?
	$u=b,\ \ \ w=c$	$Y_v:Z + Y)X_t = X_{v,t}:Z$	*Bokardo* III.

I can easily imagine the amusement which I shall afford, to some of those who can readily take in the preceding, by seriously setting myself to answer an imputation founded on such a pretext. I have no objection to help any one to a laugh : if I had, I should beg it might be at the expense of my accuser ; as it is, I am quite willing to go halves. But, I repeat, the learning and moral worth of my accuser are the amber in which this straw would be preserved, if I did not take steps to prevent it : and, for once at least, whatever it is worth his while to assert, it cannot be totally, and beyond doubt, safe for me to neglect.

There are two questions ;—First, did Sir William Hamilton communicate anything ? Secondly, allowing hypothetically that he did, was it anything more than I had proved my independent right to when I sent my paper to Cambridge ? He says (November 2) that the requisites will " show the nature of his doctrine of syllogism, in one of its halves," and again (March 13) that he communicated " the fundamental doctrine." And (April 13) he speaks of a doctrine founded on the *expressed quantity* of the predicate, as the thing in question. It is therefore the *second* requisite on which this part of the case turns. Now no predecessor of mine that I know of ever *expressed* or *understood* any quantity in a logical assertion except *none*, the indefinite *some* of the particular proposition, and *all*. Sir William never hints at *defining* the quantity of the terms *in particular propositions*, on which the whole of my second system turns. And it is for him to prove, both (as I shall presently remark) that he and I mean the same thing by *quantity*, and also that he has produced to his class a syllogism of *particular* premises with *definite* quantities. When

the *Requisites* above cited first reached me, and I saw mention of the quantities which both subject and predicate *always have* in thought, all I could do was to wait for the *more articulate statement* which was promised me. It remained to see whether Sir William Hamilton was really speaking of what they *always have* in thought, which the common system represents, or of what they *have not but always would have if our knowledge were exact enough*, which is what my extension of it supposes.

The reader who is not used to logic will be rather surprised to learn that a person kept close to Aristotle's forms could not prove that in a company of men, most of whom have coats and most of them waistcoats, it must be that some of them have both coats *and* waistcoats. The inference is most demonstrably legitimate : but all the schoolmen that ever lived could not put it into the form of an Aristotelian syllogism. If Sir William Hamilton can prove that he ever, before his communications with me, actually exhibited such an inference to his class, or another of a like kind, then, and not till then, will he show that he so far preceded me. I will presently put the burden of proof on him more distinctly.

Next, could I, after writing what is in my paper, want anything necessary for the addition ? I will here observe, that my system in no degree depends upon giving definite quantity to the predicate, as *predicate*. It consists entirely in the quantification of the *middle term*, be it subject or predicate. In fact, perfectly definite quantification destroys the necessity of distinguishing subject and predicate. To say that some 20 Xs out of 50, are all to be found among 70 Ys, or that 20 out of 50 Xs are 20 out of 70 Ys, is precisely the same thing as saying that 20 out of 70 Ys are 20 out of 50 Xs.

Now the only distinction between the original paper and the addition is, that in the former I have been thinking of the cases of inference which Aristotle could not include, and in the latter of those which he did include : both on one principle, explicitly stated in both. In the former I have only quantified the middle term, *be it subject or predicate*, in the latter I have noted the manner in which that same canon derived from the middle term is expressed when there is the quantification of *all* terms. In the former I have expressed the quantity of my conclusion, there called *the* middle term, being as much as is really *middle*, by $m+n-1$, and Y_1+Y_2-1 : in the latter by $v+w-b$, &c. ; the resulting number instead of the resulting fraction of the whole. I am really so much at a loss to imagine what I have done in the Addition with any tool not used in the Memoir, that I must wait till Sir William Hamilton points it out, which I hope he will do very explicitly. He must, for his own character's sake, attempt to join my Addition on to the Memoir by a hook of *breach of confidence* and an eye of *false dealing*, manufactured out of the materials of his letter of November 2, and the *Requisites* which he sent with them. I hope he will not draw back, but set manfully to work, and try it : I am curious as well as interested.

In his letter of March 13, Sir William refers again and again to my having acknowledged his priority : this I must briefly set right. At every period I have given him all that I thought it likely he had. Accordingly, in my correspondence, when I knew nothing but his *results* as stated in the prospectus, I treated it as certain that we must have something in common, in which case he, as having published it in lectures, must have had undoubted priority. But when, after reading Mr. ——— ———'s notes, my impression was much weakened, I used the less confident language quoted above in the extract from my *Addition*. And now, my belief in our having anything in common being reduced to thinking it very improbable, by further search through those notes,

I tell him that I will admit such priority as he shall prove, and no more. Nor will I admit his own simple assertion : for though I have full reliance on his veracity, yet he is as liable as myself to the well-known mistake of investigators, upon whose previous views the approach of those of others frequently throws such a new and sudden light, that they think they always must have seen that which they do then see. In my case I contend that *others* will see first, that he never did open his views ; secondly, that if he did, any one (not myself merely) can see that I had nothing to gain by it. And to others I submit those points.

I now come, in the last place, to what would have been the previous question, if this statement had been purely literary. It is this, is or is not Sir William Hamilton under a mistake in supposing that he ever used the term *quantification* in the sense in which I use it ? On this point, I will, as I said, bring forward enough to throw the onus of proof upon him.

First, even in the Prospectus, *definite* quantity is divided into *universal* and *singular* ; *all*, and *one*. No hint of any proposition in which *half*, or *more than half*, or *two-thirds*, &c. are mentioned. In fact, whereas the old logi- cians made the *singular* proposition count as *universal*, Sir William makes a distinction between *universal* (all, when there are more than one,) and *singular*, (all, when there is only one,) and calls them both *definite*, and the particular proposition *indefinite*, thereby at once distinguishing his system from mine, the very essence of which is the consideration of *definite particulars*. Had I taken notice of this when I first saw the Prospectus, I should not have sup- posed it likely we had anything in common. Sir William Hamilton cannot even maintain his *claim* against me, much less his *charge*, without directly contradicting the very Prospectus which he published " formally to establish " his " right of authorship."

The passage from the Prospectus is as follows :

> " That the *preindesignate terms* of a proposition, whether subject or predicate, are never, on that account, thought as *indefinite* (or indeterminate) in quantity. The only indefinite, is *particular*, as opposed to *definite*, quantity ; and this last, as it is either of an extensive *maximum* undivided, or of an extensive *minimum* indivisible, constitutes quantity *universal* (general), and quantity *singular* (individual). In fact, *definite* and *indefinite* are the only quantities of which we ought to hear in Logic ; for it is only as indefinite that particular, it is only as definite that individual and general, quantities have any (and the same) logical avail."

When Sir William Hamilton mentioned to me that he had requested Mr. ———————— to give me information on his system, he described that gentle- man as having obtained one of his highest class honours, and therefore fully competent to afford me information. And further, that various topics (page 4), among which was quantification of the predicate, were discussed in the class, partly given out as exercises, and " in fact mere common-place." Mr. ———————— (who was my own pupil, and to whose intelligence I add my testimony) lent me his notes, full and clear, and with a table of contents. Surely then, there must be something in these notes to show that Sir William Hamilton had preceded me. *All I can say is I can find nothing.* I can find no notion of *quantity* except the old one of *plurality of attributes* and of *genus* and *difference*. I can find no notion whatever of *numerically defi- nite* quantity. I cannot find one single inference that is not obtained in the old system. I cannot find one single syllogism in which the middle term is not universal in one premiss. Mind, I say this simply to put the proof upon Sir William Hamilton. I do not deny that such things may be there : I cannot find them. I do not deny (and, when I see it, shall admit) that *other* notes may supply what is not in these : all I know is that *these* notes

are Sir William's chosen evidence, and I cannot find anything in them which seems to me at all connected with the points on which I ventured to propose a departure from the established system. There is much which is new to me, and much which I shall study with great interest when Sir William Hamilton's treatise appears : in particular, the manner in which life is given to the (as I once thought it) useless distinction of figures. But any consideration of definite particulars, any inference which is not an Aristotelian syllogism, is wholly wanting, so far as I can see after a very attentive examination.

But if there really be anything in which Sir William Hamilton has pre-ceded me, I shall be, of all men except himself, most interested in his having his full rights. And I make him this offer, and will take his acceptance of it as reparation in full for his suspicions and assertions. With the consent of the gentleman to whom these notes belong, which I am sure will not be re-fused to our joint application, I will forward to him a copy of their table of contents, having more than a hundred and fifty headings. From these Sir William Hamilton shall select those which are, in his opinion, sure to con-tain proof of his priority on any point which I have investigated. Of these I will have copies made and sent to him : and will print in the work on Logic which I am preparing (and in some one part of it) the parts which he shall select as fit to prove (or to show that he could prove, let him call it as he likes) his case, or the germs of his case (as he pleases, again). Provided always, that the matter shall not run beyond some eight or a dozen octavo pages of small print. And I on my part propose that I shall be allowed to print, to one-half the amount selected by Sir William Hamilton, of additional extract : but if this be refused I will not insist on it. With this I will put a heading fully descriptive of the reason and meaning of the insertion, and such distinct reference and account at the beginning of the preface as shall be sure to call the reader's attention to it. So that my book shall establish the claim, if it can be established from the notes of one of the best students. If this offer be not accepted, an account of it will take the place of any other re-sult. If Sir William Hamilton, or any one else, can propose anything to make this offer fairer, I shall probably not be found indisposed to accept the addition. And though, I will frankly say, my present conviction is that the acceptance of the offer would alone cause my work to knock Sir William Hamilton's assertions to atoms, yet I will pledge myself, in any case, to abide by it.

And now I have done with the matter, at least until Sir William Hamilton shall have had his turn. And I trust this will speedily come about ; for lengthened controversies are nuisances to the parties engaged, and would be to others, if there did not exist that plenary power of not reading which makes apologies for publication superfluous.

A. DE MORGAN.

University College, London,
April 27, 1847.